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Quantitative hazard assessment system (Has-Q) for open pit mine slopes

Tatiana Barreto dos Santos^a, Milene Sabino Lana^b, Tiago Martins Pereira^c, Ismet Canbulat^{d,*}

^a Mining Engineering Department, Universidade Estadual de Minas Gerais, João Monlevade 35930314, Brazil

^b Mining Engineering Department, Universidade Federal de Ouro Preto, Ouro Preto 35400000, Brazil

^c Statistical Department, Universidade Federal de Ouro Preto, Ouro Preto 35400000, Brazil

^d School of Minerals and Energy Resources Engineering, University of New South Wales, Sydney, NSW 2052, Australia

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ABSTRACT

Rock slope hazard assessment is an important part of risk analysis for open pit mines. The main parameters that can lead to rock slope failures are the parameters traditionally used in geomechanical classifications, the slope geometrical parameters and external factors like rainfall and blasting. This paper presents a methodology for a hazard assessment system for open pit mine slopes based on 88 cases collected around the world using principal components analysis, discriminant analysis and confidence ellipses. The historical cases used in this study included copper, gold, iron, diamond, lead and zinc, platinum and claystone mines. The variables used in the assessment methodology are uniaxial compressive strength of intact rock; spacing, persistence, opening, roughness, infilling and orientation of the main discontinuity set; weathering of the rock mass; groundwater; blasting method; and height and inclination of the pit. While principal component analysis was used to quantify the data, the discriminant analysis was used to establish a rule to classify new slopes about its stability condition. To provide a practical hazard assessment system, confidence ellipses were used to propose a hazard graph and generate the HAS-Q. The discriminant rule developed in this research has a high discrimination capacity with an error rate of 11.36%.

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1. Introduction

Standards Australia defines hazard as a source of potential harm; and a potential occurrence or condition that could lead to injury, damage to the environment, delay or economic loss [1]. In open pit slopes, potential harm is directly related to the occurrence and the volume of material in the slope failure. Three scales of slope failures can occur in an open pit mine: bench failure, inter-ramp failure and overall failure (Fig. 1).

These three types of scale of slope failures are directly related to the volume of the slope failure and, therefore, related to the consequences of these failures. Once the volume of failures is related to the consequences of these failures, the levels of acceptance criteria in probabilistic stability analysis vary according to the severity of the failure. Swan et al. provide recommendations of acceptable levels of probability of failure in open pit slopes [2,3]. The acceptance criterion of the probability of failure in open pit mines, proposed by the authors, is directly related to the consequences of the failures. The greater the failure consequences, the lower the level of tolerance accepted.

Bench failures have less serious consequences. In general, this type of failures have a minimum economic impact on production, mostly related to cleanup costs. Damage to equipment and injuries to personnel are unlikely provided that the failure does not occur when the slope is under construction [4]. Swan and Sepulveda state that bench failures are inevitable and permissible provided the acceptable contained volumes of material on benches are unlikely to be exceeded [2]. Nonetheless, benches located immediately above and below ramps and those in the final wall must have lower tolerances of failure compared to the other benches. Priest and Brown also suggest that the consequences of failures in individual benches, temporary slopes and benches that are not adjacent to haulage roads are not largely serious [3].

The consequences of inter-ramp failures are more significant than the bench failures. Injuries to personnel and damage to equipment are likely. The economic impact on production is also more significant as the production losses and cleanup costs are usually greater than the bench failures [4]. Acceptance of inter-ramp instabilities depends on the amount of ramp loss and the overall volume of material in the failure [2]. Priest and Brown suggest that failures in medium sized slopes of 50 to 100 m in height, with haulage roads or close to permanent mine installations may have serious consequences [3].

* Corresponding author.

E-mail address: i.canbulat@unsw.edu.au (I. Canbulat).

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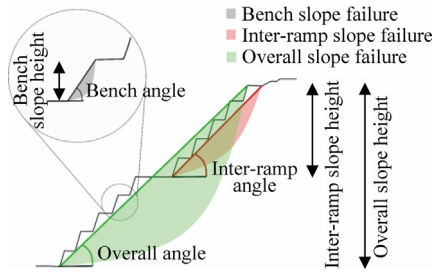


Fig. 1. Three types of slope failures in open pit mines.

The most severe case of failure is the overall slope failure. Injuries and fatalities to personnel and damage to equipment are highly likely to be serious if failure occurs in working hours. The economic impacts may be irreversible because failure can lead to ore dilution and consequently, a decrease in the ore's economic value. Finally, public and stakeholder relationships may be severely affected and may even lead to the loss of permission to mine [4]. Swan et al. state that failures in high slopes larger than 150 m height have very serious consequences and the acceptance criterion of probability is the least permissive [2,3].

Practitioners use probabilistic analysis to calculate the likelihood and consequence of a failure. However, probabilistic analyses require extensive distribution of all input parameters. This paper presents a nonparametric hazard assessment system, HAS-q, which could be applied to any open pit slope. This system is a user-friendly, relatively easy to use in engineering practice, like methodologies for risk assessment in coal mines proposed by Vardar et al. [5–7]. The HAS-q was developed by applying multivariate statistical techniques including principal component analysis, discriminant analysis and confidence ellipses. These techniques provide the quantitative hazard assessment system, since they allow calculation of the failure susceptibility level.

2. Material and methods

2.1. Database

A database containing 88 slopes was constructed based on case studies around the world, using published papers and books which

encompass many worldwide open pit slope stability case histories. Based on 84 slopes of these case studies, Naghadehi et al. proposed a mine stability index for mine slopes [8]. These 84 slopes were used in this research. An additional four cases have been collated from a Brazilian mine. Table 1 presents the mines used to build the database.

The database was built using the variables chosen by the authors to develop the hazard system and the variables considered were those traditionally used in rock mass classification systems and the open pit slope geometry parameters, including the uniaxial compressive strength of intact rock (P_1), spacing of discontinuities (P_2), persistence of discontinuities (P_3), opening (P_4), roughness (P_5), infilling (P_6) and orientation of the main discontinuity set (P_7), weathering of the rock mass (P_8), groundwater (P_9), blasting method (P_{10}), height (P_{11}), and inclination (P_{12}) of the pit.

The database was built using the values presented in Tables 2–8. For quantitative variables, see Table 2, the original values of the parameters were used. For qualitative variables, ordinal values were assigned ranging from 1 to 5, see Tables 3–8. The stability status of each slope in the database is known: stable (ST), bench and inter-ramp failure (FSB) and overall failure (OF).

Here, α_d is the dip direction of the main discontinuity; α_s is the dip direction of the slope; β_d is the main discontinuity dip; and β_s is the slope dip.

2.2. Multivariate analysis techniques

Multivariate analysis technique is used to create knowledge and thereby improve the decision making. Multivariate analysis refers to all statistical techniques that simultaneously analyze multiple measurements on individuals or objects under investigation. An

Table 2
Quantitative variables.

| Variable | Parameter |
|--|-----------|
| Uniaxial compressive strength (UCS) of intact rock (MPa) | P_1 |
| Spacing of the main discontinuity set (m) | P_2 |
| Persistence of the main discontinuity set (m) | P_3 |
| Aperture of the main discontinuity set (mm) | P_4 |
| Pit height (m) | P_{11} |
| Pit inclination ($^\circ$) | P_{12} |

Table 1
Mines used in the database.

| Mine | Country | Ore | Number of slopes in each mine (stable and unstable slopes) |
|--------------|------------------|-----------------|--|
| Aguas Claras | Brazil | Iron | 5 |
| Aitik | Sweden | Copper | 6 |
| Alegria | Brazil | Iron | 4 |
| Angooran | Iran | Lead and zinc | 4 |
| Aznalcollar | Spain | Lead and zinc | 5 |
| Betze-Post | USA | Gold | 4 |
| Cadia Hill | Australia | Gold and copper | 5 |
| Chadormalou | Iran | Iron | 5 |
| Choghart | Iran | Iron | 5 |
| Chuquicamata | Chile | Copper | 5 |
| Escondida | Chile | Copper | 7 |
| Esperanza | USA | Copper | 1 |
| Gole-Gohar | Iran | Iron | 4 |
| La Yesa | Spain | Claystone | 2 |
| Ok Tedi | Papua New Guinea | Gold and copper | 2 |
| Panda | Canada | Diamond | 1 |
| Sandsloot | South Africa | Platinum | 6 |
| Sarcheshmeh | Iran | Copper | 4 |
| Sungun | Iran | Copper | 5 |
| Ujina | Chile | Copper | 1 |
| Venetia | South Africa | Diamond | 7 |

Table 3

Roughness of the main discontinuity set–P5 [9].

| Description | Very rough | Rough | Slightly rough | Smooth | Slickensided |
|-------------|------------|-------|----------------|--------|--------------|
| Rating | 5 | 4 | 3 | 2 | 1 |

Table 4

Infilling of the main discontinuity set (adapted from Bieniawski)–P6 [6].

| Description | None | Hard filling <5 mm | Hard filling >5 mm | Soft filling <5 mm | Soft filling >5 mm |
|-------------|------|--------------------|--------------------|--------------------|--------------------|
| Rating | 5 | 4 | 3 | 2 | 1 |

Table 5

Weathering of the rock mass (International Society of Rock Mechanics)–P7 [10].

| Description | Fresh (W1) | Slightly weathered (W2) | Moderately weathered (W3) | Highly weathered (W4) | Completely weathered (W5) |
|-------------|------------|-------------------------|---------------------------|-----------------------|---------------------------|
| Rating | 5 | 4 | 3 | 2 | 1 |

Table 6

Groundwater (adapted from Bieniawski)–P8 [9].

| Description | Completely dry | Damp | Wet | Dripping | Flowing |
|-------------|----------------|------|-----|----------|---------|
| Rating | 5 | 4 | 3 | 2 | 1 |

Table 7

Orientation of the main discontinuity set (adapted from Naghadehi et al.)–P9 [8].

| Description | $\beta d > \beta s$ $\alpha d - \alpha s > 30^\circ$ Very favourable | $\beta d > \beta s$ $\alpha d - \alpha s < 30^\circ$ Favourable | $0 \leq \beta d \leq \frac{\beta s}{4}$ $\alpha d - \alpha s > 30^\circ$ Fair | $\frac{\beta s}{4} \leq \beta d \leq \frac{\beta s}{2}$ $\alpha d - \alpha s < 30^\circ$ Unfavourable | $\frac{\beta s}{2} \leq \beta d \leq \beta s$ $\alpha d - \alpha s < 30^\circ$ Very unfavourable |
|-------------|--|---|---|---|--|
| Rating | 5 | 4 | 3 | 2 | 1 |

Table 8

Blasting method (adapted from Naghadehi et al.)–P10 [8].

| Description | Presplitting | Postsplitting | Smooth wall/cushion | Modified production blast | Regular blasting/mechanical |
|-------------|--------------|---------------|---------------------|---------------------------|-----------------------------|
| Rating | 5 | 4 | 3 | 2 | 1 |

analysis, to be considered truly multivariate, all variables must be random and correlated in a such way that the purpose of different effects cannot meaningfully be interpreted separately [11].

To verify if the database used in multivariate analysis present significant correlation between the variables, Bartlett's testis used [12].

2.2.1. Principal component analysis

Principal component analysis is a multivariate statistical technique that uses an orthogonal transformation to convert a set of observations of correlated variables into uncorrelated variables named principal components [13]. The new variables (principal components) are linear combinations of the p variables of the original data set, see Eq. (1).

$$PC_i = e_i^t X = e_{i1}X_1 + e_{i2}X_2 + e_{i3}X_3 + \dots + e_{ip}X_p \quad (1)$$

where PC_i is the principal component i , $i = 1, 2, \dots, p$; e_i^t the transposed eigenvector i of the correlation matrix of the data; and X the vector of original variables.

The variance of each principal component is equal to the eigenvalue related to the eigenvector of that component. Therefore, the proportion of total variance of the original data that is explained for the i^{th} principal component is calculated by Eq. (2).

$$P_i = \frac{\lambda_i}{\sum_{i=1}^p \lambda_i} \quad (2)$$

where P_i is the proportion of total variance explained for the i^{th} principal component; and p the number of variables and λ_i the i^{th} eigenvalue.

Principal component analysis is a technique that is often used to quantify data with qualitative variables. Then, it is applied previously to techniques that can only be applied in quantitative data, e.g., discriminant analysis.

2.2.2. Discriminant analysis

Discriminant analysis is a multivariate technique used for classifying the elements of a sample in different populations. The technique must to be applied only in quantitative data. The classification rule is built using a function able to distinguish between two or more groups through original features that must be known for all the groups. Knowledge of the populations allows the formulation of a discrimination rule which can be used to classify new individuals.

Fisher's canonical discriminant functions consist of linear combinations \hat{Y}_j , see Eq. (3) [14].

$$\hat{Y}_j = \hat{e}_j^t X_{px1} \quad j = 1, 2, \dots, s \leq \min(k-1, p) \quad (3)$$

where \hat{e}_j is the j^{th} eigenvector that corresponds to the j^{th} greater eigenvalue of the matrix $W^{-1}B$; X the vector of variables; W the matrix of squares and cross products within the groups (Eq. (4)); and B the matrix of squares and cross products between groups (Eq. (5)). They are calculated as follows:

$$W_{p \times p} = \sum_{i=1}^k \sum_{b=1}^{n_i} (X_{ib} - \bar{X}_i)(X_{ib} - \bar{X}_i)^t \quad (4)$$

$$B_{p \times p} = \sum_{i=1}^k n_i (\bar{X}_i - \bar{X})(\bar{X}_i - \bar{X})^t \quad (5)$$

where p is the number of variables; k the number of populations; n_i the number of elements in the class i ; X_{ib} the vector of observations of the sample element b which belongs to population i ; \bar{X}_i the vector of means for i^{th} population; and \bar{X} the vector of means for the total sample.

For each individual, a vector \hat{Y}_j is calculated. The scores of canonical discriminant functions applied to the vector of means for each class are also calculated (\hat{Y}_i). Then, the Euclidean distance between \hat{Y}_j and \hat{Y}_i is calculated through Eq. (6). Finally, individuals are classified in the population whose Euclidean distance is smaller.

$$d = \sqrt{(\hat{Y}_j - \hat{Y}_i)^t (\hat{Y}_j - \hat{Y}_i)} \quad (6)$$

Fisher's canonical discriminant functions do not depend intrinsically on the multivariate normality of the data.

2.3. Confidence ellipses

Mahalanobis distance between a point and a vector of the means of a distribution in the bivariate space is calculated by Eq. (7) [15]. Mahalanobis distance is different from the traditional Euclidean distance as it takes into account the covariance between the data.

$$d^2(x, \bar{x}) = (x - \bar{x})^T \Sigma^{-1} (x - \bar{x}) \quad (7)$$

where $d^2(x, \mu)$ is the Mahalanobis distance from a point to the vector of means; $\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$ the vector of means of the variables x_1 and x_2 ; and $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$ the covariance matrix of x_1 and x_2 .

The confidence ellipses delineate the points whose Mahalanobis distances are equal or smaller than the axes of the ellipse [11]. Choosing a suitable significance level value α , ellipses containing $(1 - \alpha)\%$ of the points in a given data set can be generated (Fig. 2). Eq. (8) presents an equation of an ellipse centred on the vector of means (\bar{x}) of variables x_1 and x_2 with axes in the direction of eigenvectors of the covariance matrix (S). The lengths of the axes

are proportional to the respective eigenvalues of the covariance matrix.

If a bivariate normal distribution of x_1 and x_2 can be assumed, the Mahalanobis distance from a point to the vector of means follows a chi-square distribution (χ^2) with two degrees of freedom and confidence ellipses can be obtained by Eq. (8).

$$(X - \bar{x})^t \Sigma^{-1} (X - \bar{x}) \leq \chi_2^2(\alpha) \quad (8)$$

The methodology was implemented in the statistical software *R* and results reported in the next section [16].

3. Results and discussion

3.1. Bartlett's test

The Bartlett's test was used to verify if the database is suitable for the application of multivariate statistical techniques [12]. The test consists of the comparison of the correlation matrix of the p variables to the identity matrix. The test is shown in Table 9. Once the p-value approaches zero, there is sufficient correlation between the data and the database is suitable for application of multivariate statistical techniques.

3.2. Principal component analysis

Principal component analysis was used to quantify the data. It is a prerequisite to apply discriminant analysis when the database has some qualitative variables.

As the original data has 12 parameters, 12 principal components were generated. Table 10 presents the proportion of variance explained by each of the principal components and Table 11 presents the coefficients of the 12 linear combinations generated by the principal analysis technique.

Principal components analysis yielded 12 quantitative scores for each of the 88 slopes which were used to provide the classification rule through discriminant analysis.

3.3. Discriminant analysis

The classification rule was built for three populations (statuses: stable (ST), inter-ramp and bench failures (FSB) and overall failures (OF)) using Fisher's canonical discriminant functions [14]. These discriminant functions are normally used for homoscedastic data, i.e., data with the same population covariance matrices. Nevertheless, when the multivariate normality hypothesis cannot be assumed, Fisher's canonical discriminant functions are the preferred option, since quadratic discriminant analysis presents sensitivity to normality. Box's M Test was used to test the homoscedasticity population covariance matrix [17].

Box's M test and multivariate normality test were carried out and the results are presented in Tables 13 and 14.

As the p-value of Box's M test is smaller than 0.05, it is assumed that the data is not homoscedastic, making quadratic discriminant analysis an alternative. However, the p-value of the normality test approaches zero and, therefore, the multivariate normality hypothesis cannot be assumed. As quadratic discriminant analysis

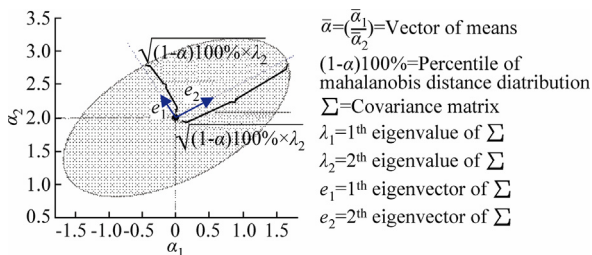


Fig. 2. Confidence ellipse.

Table 9
Bartlett's test results.

| Parameter | Value |
|-----------|------------------------|
| χ^2 | 271.49 |
| df | 66 |
| p-value | 9.77×10^{-27} |

Table 10

Proportion of variance explained by each of the principal components.

| Item | PC ₁ | PC ₂ | PC ₃ | PC ₄ | PC ₅ | PC ₆ |
|----------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Proportion of explained variance | 0.2335 | 0.1568 | 0.1443 | 0.1118 | 0.0809 | 0.0659 |
| Cumulative proportion | 0.2335 | 0.3903 | 0.5346 | 0.6464 | 0.7273 | 0.7932 |

Table 11

Proportion of variance explained by each of the principal components.

| | PC ₇ | PC ₈ | PC ₉ | PC ₁₀ | PC ₁₁ | PC ₁₂ |
|----------------------------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|
| Proportion of explained variance | 0.0492 | 0.0434 | 0.0375 | 0.0292 | 0.0250 | 0.0225 |
| Cumulative proportion | 0.8424 | 0.8858 | 0.9233 | 0.9525 | 0.9775 | 1 |

Table 12

Coefficients of the 12 equations generated by principal component analysis.

| PC _i | e _{i1} | e _{i2} | e _{i3} | e _{i4} | e _{i5} | e _{i6} | e _{i7} | e _{i8} | e _{i9} | e _{i10} | e _{i11} | e _{i12} |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|
| 2 | 0.35 | -0.06 | -0.23 | 0.12 | -0.08 | 0.05 | -0.07 | -0.42 | 0.30 | 0.55 | 0.06 | 0.48 |
| 3 | -0.07 | 0.47 | 0.28 | 0.26 | 0.08 | -0.33 | 0.19 | 0.00 | 0.28 | 0.19 | 0.57 | -0.20 |
| 4 | -0.40 | 0.12 | -0.29 | -0.17 | 0.05 | 0.46 | -0.30 | -0.32 | -0.21 | 0.19 | 0.36 | -0.29 |
| 5 | -0.15 | 0.06 | -0.21 | -0.61 | -0.62 | -0.29 | 0.15 | 0.05 | 0.23 | 0.08 | 0.01 | -0.07 |
| 6 | -0.23 | -0.14 | -0.40 | 0.06 | 0.37 | -0.69 | -0.32 | 0.00 | -0.08 | 0.15 | -0.12 | -0.08 |
| 7 | -0.23 | 0.57 | 0.13 | -0.14 | 0.01 | -0.08 | -0.33 | -0.09 | -0.10 | -0.28 | -0.07 | 0.61 |
| 8 | 0.18 | 0.45 | -0.39 | 0.17 | 0.01 | 0.21 | -0.22 | 0.25 | 0.47 | -0.09 | -0.34 | -0.29 |
| 9 | 0.59 | -0.10 | 0.10 | -0.25 | -0.06 | -0.10 | -0.58 | 0.12 | -0.06 | -0.17 | 0.40 | -0.11 |
| 10 | -0.29 | -0.28 | 0.20 | -0.31 | 0.37 | 0.20 | -0.14 | 0.40 | 0.52 | 0.15 | 0.11 | 0.21 |
| 11 | -0.17 | -0.16 | -0.39 | 0.44 | -0.34 | 0.06 | -0.02 | 0.46 | -0.08 | -0.14 | 0.39 | 0.31 |
| 12 | -0.06 | 0.13 | 0.33 | 0.10 | -0.26 | 0.03 | -0.28 | 0.38 | -0.32 | 0.63 | -0.26 | -0.06 |

Table 13

Box's M test results.

| Parameter | Value |
|-----------|--------|
| χ^2 | 207.04 |
| df | 156 |
| p-value | 0.0039 |

Table 14

Multivariate normality test results.

| Parameter | Value |
|-----------|-----------------------|
| H | 20.76 |
| p-value | 1.24×10^{-5} |

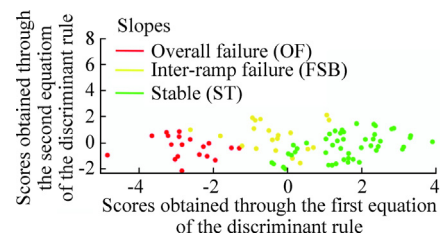
requires normality, Fisher's canonical discriminant functions were used.

As the database has three populations, the classification rule has two discriminant functions. After performing discriminant analysis, the discriminant scores of the 88 slopes were calculated and these scores were graphed. The scores of the first 12 principal components plus the status of each slope make up the sample for the discriminant analysis.

After applying Fisher's canonical discriminant functions, the classification rule was obtained as follows Eqs. (9) and (10).

$$\hat{Y}_1 = -1.00PC_1 - 0.75PC_2 - 0.23PC_3 - 0.19PC_4 - 0.29PC_5 - 0.11PC_6 - 0.15PC_7 - 0.08PC_8 - 0.02PC_9 + 0.39PC_{10} + 0.32PC_{11} + 0.31PC_{12} \quad (9)$$

$$\hat{Y}_2 = 0.26PC_1 - 0.01PC_2 - 0.39PC_3 + 0.09PC_4 - 0.03PC_5 + 0.36PC_6 + 0.07PC_7 - 0.51PC_8 + 0.37PC_9 - 0.20PC_{10} - 0.05PC_{11} + 0.01PC_{12} \quad (10)$$

**Fig. 3.** Scores of the 88 slopes in discriminant analysis.

where \hat{Y}_1 is the first equation of the discriminant rule; \hat{Y}_2 the second equation of the discriminant rule; and PC_i , $i = 1, 2, \dots, 12$ the i^{th} principal component score.

The discriminant scores, obtained through Eqs. (9) and (10), for the 88 slopes in the database were calculated and presented in Fig. 3, which shows a clustering tendency of the scores in the three statuses of the slopes. Therefore, the results yielded by discriminant analysis are in good agreement with the actual information available about the slopes.

3.4. Validation

The resubstitution method was used to validate the classification rule generated through discriminant analysis. In this method, the total sample is used to generate the classification rule. Then, the predicted class of the total sample of individuals obtained through comparison of the classification rule to the actual class. When the classification rule is of good quality a low error rate is expected. The apparent error rate is calculated using the validation matrix (Table 15) and Eq. (11).

$$AER = \frac{n_{12} + n_{13} + n_{21} + n_{23} + n_{31} + n_{32}}{n_1 + n_2 + n_3} \quad (11)$$

Table 15

Validation matrix.

| Item | Predicted class | | | |
|--------------|-----------------------|----------|----------|----------|
| | Populations (classes) | ST | FSB | OF |
| Actual class | ST | n_{11} | n_{12} | n_{13} |
| | FSB | n_{21} | n_{22} | n_{23} |
| | OF | n_{31} | n_{32} | n_{33} |

Table 16

Validation matrix of 88 slopes.

| Item | Predicted class | | | | |
|--------------|-----------------|-----|----|----|-------|
| | Class | FSB | OF | ST | Total |
| Actual class | FSB | 19 | 2 | 0 | 21 |
| | OF | 2 | 17 | 0 | 19 |
| | ST | 6 | 0 | 42 | 48 |
| | Total | 27 | 19 | 42 | 88 |

where AER is the apparent error rate; n_{ij} the number of slopes in the test sample that were classified in class i but are from class j ; and n_i the sample size of class i .

Table 16 presents the classifications of the 88 slopes by discriminant analysis and their actual classes.

Only two slopes that actually presented a more serious failure (OF) were classified in a less serious category (FSB). The other 8 slopes that actually presented less serious failures were classified in more serious categories: being 2 slopes that presented overall failures (OF) were classified as inter-ramp failures (FSB) and 6 slopes that were stable (ST) were classified as inter-ramp failures (FSB) by the classification rule. Therefore, the classification rule can be considered conservative. Nevertheless, the overall error rate of the classification rule is 11.36%, which is believed to be acceptable since the geotechnical parameters used include a number of uncertainties due to the variability of rock masses.

3.5. Construction of the hazard graph

Three confidence ellipses for the three populations (FSB, OF and ST) were used to construct the hazard assessment graph. Mahalanobis distance was used to generate confidence ellipses of the distance distributions of the scores of the 88 slopes obtained through the two equations generated by discriminant analysis.

The first step was check if the discriminant scores of the three populations present a bivariate normal distribution. Table 17 presents the results of the bivariate normality test for the three populations.

The P-values of the normality tests for the three populations are significant and, therefore, the multivariate normality hypothesis can be assumed and the Eq. (8) can be used to obtain the confidence ellipses.

Three confidence ellipses of discriminant scores were constructed for the three populations to determine the limits that separate the three classes. Then, the border between each class was

Table 17

Bivariate normality test for FSB, OF and ST populations.

| Population | Parameter | Value |
|------------|-----------|-------|
| FSB | H | 0.84 |
| | p-value | 0.65 |
| OF | H | 3.06 |
| | p-value | 0.22 |
| ST | H | 0.89 |
| | p-value | 0.64 |

drawn from the intersection points between the confidence ellipses. Significance levels (α) of 0.2, 0.1 and 0.05 were considered in the construction of ellipses (Fig. 4).

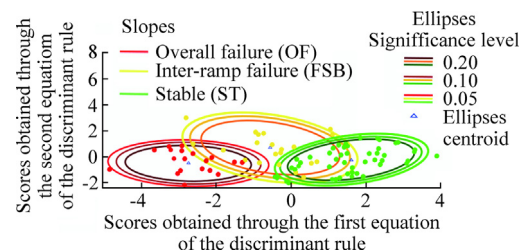
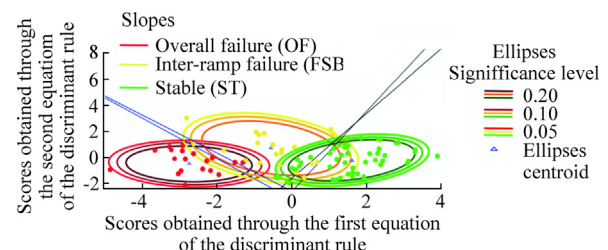
Using the limits that separate the three populations (OF, FSB and ST) a hazard assessment system to classify new mine slopes is proposed.

To identify if different levels of significance would change the location of the limits between the classes, the boundaries were drawn with levels of significance equal to 0.2 and 0.05, shown in Fig. 5. The boundaries obtained through ellipses with levels of confidence equal to 0.2 and 0.05 have almost the same location. As a result, the confidence ellipses with the level of significance equal to 0.05 were used to construct the hazard assessment system.

Eqs. (12), (13) and (14) are the equations of the confidence ellipses with confidence level equal to 0.05 for the OF, FSB and ST classes, respectively.

$$0.99\hat{Y}_1^2 + 1.45\hat{Y}_2^2 + 5.55\hat{Y}_1 + 1.96\hat{Y}_2 + 0.19\hat{Y}_1\hat{Y}_2 = -3.54 \quad (12)$$

$$1.00\hat{Y}_1^2 + 0.75\hat{Y}_2^2 + 0.73\hat{Y}_1 - 0.75\hat{Y}_2 + 0.56\hat{Y}_1\hat{Y}_2 = 4.51 \quad (13)$$

**Fig. 4.** Confidence ellipses considering the three levels of significance.**Fig. 5.** Boundaries defined with confidence ellipses with 0.2 and 0.05 of level of significance.

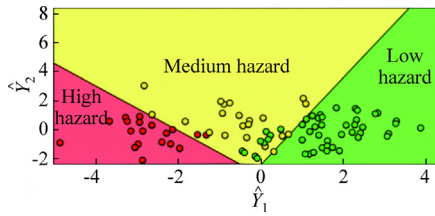


Fig. 6. Hazard assessment graph for HAS-q.

$$1.22\hat{Y}_1^2 + 1.35\hat{Y}_2^2 - 4.12\hat{Y}_1 + 2.08\hat{Y}_2 - 0.85\hat{Y}_1\hat{Y}_2 = 1.05 \quad (14)$$

The intersection points of the confidence ellipses were determined and the equations of straight lines, which contain these points, were calculated. The intersection points of the OF and FSB ellipses were $(-2.91, 1.28)$ and $(-1.01, -1.66)$; the equation of the line containing these points is given by Eq. (15). The intersection points of the FSB and ST ellipses were $(1.36, 1.49)$ and $(0.09, -2.00)$; the equation of the line containing these points is given by Eq. (16).

$$\hat{Y}_2 = -1.55\hat{Y}_1 - 3.21 \quad (15)$$

$$\hat{Y}_2 = 2.76\hat{Y}_1 - 2.26 \quad (16)$$

The potential harm of open pit slope failures is directly related to the scale of failure. The consequences of bench and inter-ramp failures are much less significant than for overall failures. Hence, the region of overall failure was named “high hazard zone”, the region of bench and inter-ramp failures was named “medium hazard zone” and the stable zone was named “low hazard zone”. Although stable slopes do not present an immediate hazard, they were considered “low hazard zone” because of the error rate of the classification rule. Fig. 6 presents the proposed quantitative hazard assessment system (HAS-q).

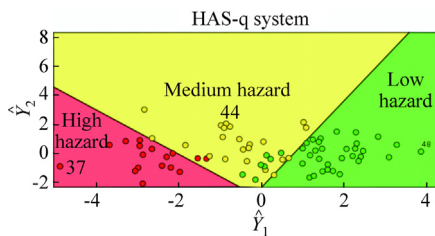


Fig. 7. 37th, 44th and 48th slopes of the data.

Table 18

Parameters of the 37th, 44th and 48th slopes of the data.

| Slope | Parameter | | | | | |
|-----------|-----------|------|------|------|-----|-----|
| 37 OF | P1 | P2 | P3 | P4 | P5 | P6 |
| | 15 | 2.50 | 7.25 | 3 | 2 | 2 |
| | P7 | P8 | P9 | P10 | P11 | P12 |
| 44 FSB | 2 | 2 | 4 | 2 | 20 | 48 |
| | P1 | P2 | P3 | P4 | P5 | P6 |
| | 180 | 1 | 22.5 | 3 | 3 | 3 |
| 48 ST | P7 | P8 | P9 | P10 | P11 | P12 |
| | 4 | 3 | 5 | 5 | 325 | 58 |
| | P1 | P2 | P3 | P4 | P5 | P6 |
| | 150 | 3 | 1.75 | 0.55 | 4 | 4 |
| | P7 | P8 | P9 | P10 | P11 | P12 |
| | 4 | 5 | 2 | 5 | 325 | 51 |

3.6. Evaluation of the hazard assessment graph and application on a new slope

Fig. 7 and Table 18 present three slopes and their parameters. Each one presents a different status. All the parameters were analyzed and present a geotechnical meaning. The four parameters that presented more interesting results are presented.

The first parameter P1 (uniaxial compressive strength) and P7 (weathering of the rock mass) are related with the strength of the rock. Overall failures are related with the strength of the rock and inter-ramp failures are related with the strength of the discontinuities. The slope with overall failure presented a very low value of uniaxial compressive strength (15 MPa) and it is highly weathered. The slope with inter-ramp failure and the stable slope presented 180 and 150 MPa, respectively, and they are slightly weathered.

The importance of efficient drainage systems could be verified in the three slopes presented. The stable is slope, which is completely dry, and the slope with inter-ramp failure is wet and the slope with overall failure is dripping.

The parameter P9 is related with the orientation of the discontinuities in the rock mass. They orientation could lead to inter-ramp failures. The 44th slope (inter-ramp failure) presented a very favorable orientation for the occurrence of discontinuity failure. The stable slope presents an unfavorable orientation and the slope with overall failure presents a favorable orientation for occurrence of discontinuity failure.

Fig. 8 present the box plots of the parameters for each slope class. They confirm the analysis performed for the slopes above.

The hazard graph was applied in a mine slope for illustration. The slope is located in an iron mine of Minas Gerais State, Brazil. The geotechnical parameters of this slope is presented (Tables 19 and 20).

The first step is calculate the twelve principal components scores based on the coefficients presented in Table 12. To illustrate, Eq. (17) presents the calculus of the first principal component score. Table 21 shows the principal component scores of the new slope.

$$\begin{aligned} PC_1 &= (-0.29 \times 175) + (-0.29 \times 2) + (0.33 \times 3) + (0.33 \times 0.6) \\ &\quad + (-0.37 \times 4) + (-0.06 \times 5) + (-0.40 \times 4) + (-0.34 \times 5) \\ &\quad + (0.35 \times 3) + (-0.19 \times 5) + (-0.12 \times 150) + (-0.15 \times 45) \\ &= -63.34 \end{aligned} \quad (17)$$

The second step is to calculate the discriminant scores using the obtained values presented in Table 21 and the Eqs. (9) and (10). The values obtained for Y_1 and Y_2 were equal to 3.3 and -1.1 ,

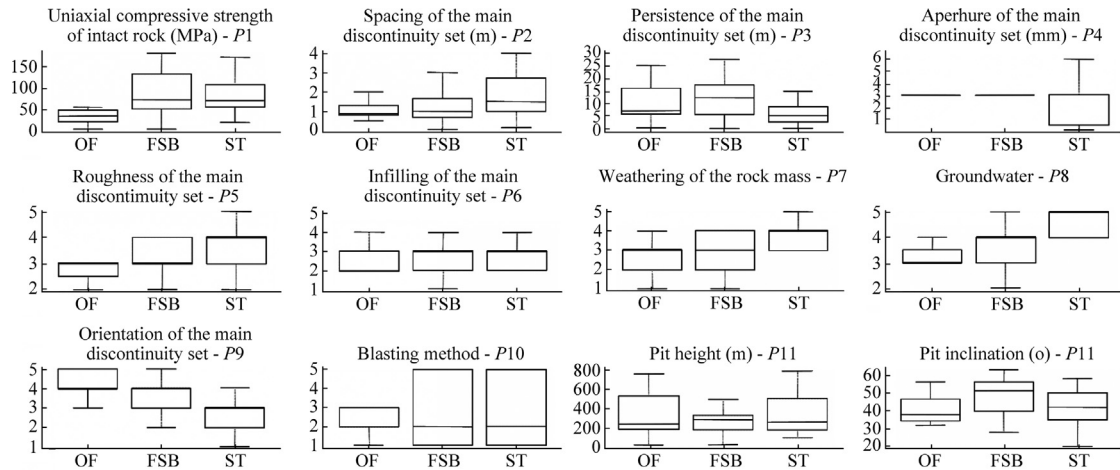


Fig. 8. Box plots of the parameters for each slope class.

Table 19

Geotechnical parameters of a Brazilian mine slope (P1-P6).

| P1 | P2 | P3 | P4 | P5 | P6 |
|----|----|----|----|----|----|
| 30 | 6 | 5 | 5 | 2 | 1 |

Table 20

Geotechnical parameters of a Brazilian mine slope (P7-P12).

| P7 | P8 | P9 | P10 | P11 | P12 |
|----|----|----|-----|-----|-----|
| 3 | 2 | 3 | 3 | 400 | 40 |

Table 21

Principal component scores of the new slope.

| | | | | | |
|-----------------|--------|-----------------|---------|------------------|---------|
| PC ₁ | -63.34 | PC ₅ | -7.09 | PC ₉ | 169.54 |
| PC ₂ | 54.18 | PC ₆ | -61.34 | PC ₁₀ | 44.8 |
| PC ₃ | 225.23 | PC ₇ | -9.5 | PC ₁₁ | 162.17 |
| PC ₄ | 117.78 | PC ₈ | -139.39 | PC ₁₂ | -104.91 |

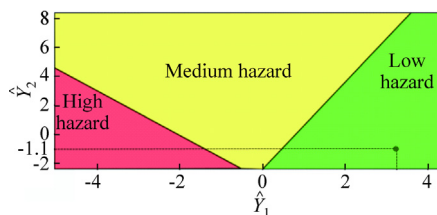


Fig. 9. Classification of the new slope.

respectively. Lastly, the hazard graph must be used for classification of the new slope (Fig. 9). The Brazilian mine slope was classified as a slope with low hazard.

4. Conclusions

The use of quantitative tools for hazard assessment in slopes is a critical part of risk management in open pit mines. Qualitative hazard assessments, which are based on the experience of geotechnical experts, are not sufficient for an adequate evaluation of geotechnical risks. Qualitative assessments should be comple-

mented by quantitative tools, avoiding the subjectivity associated with qualitative risk evaluation.

Depending on the technique used, it is possible to know the error associated with the obtained results which increases the confidence of engineers making decisions. Multivariate statistical techniques have been proven to be efficient for quantification of phenomena in many research fields as they provide quantitative methods of evaluation, in this case of the slope hazards.

Principal component analysis was used to quantify the data from the database of 88 slopes. Discriminant analysis was carried out and a discriminant rule obtained with an error rate of 11.36%. As the geotechnical parameters are inherently variable due to uncertainties in rock masses and the variability of the survey data of the other parameters, the error rate of the discriminant rule was considered reliable. Confidence ellipses were used to develop HAS-q, which can be used to assess the hazards in mine slopes. HAS-q is a user-friendly, powerful tool for hazard assessment as it provides an unbiased assessment of the hazards. Since it was developed based on 88 slopes, from copper, gold, iron, diamond, lead and zinc, platinum and claystone mines located around the world, with high variability in their geotechnical parameters, the HAS-q methodology can be used for all open pit slopes. As this is an empirical tool, future research can improve the database used to develop the HAS-q and to revise the transition zones between different hazard classes.

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